S–CALCF

NEW ZEALAND QUALIFICATIONS AUTHORITY
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Scholarship 2005
Mathematics with Calculus

FORMULAE AND TABLES BOOKLET

Refer to this booklet to answer the questions in the Question Booklet 93202Q.

Check that this booklet has pages 2–4 in the correct order and that none of these pages is blank.

YOU MAY KEEP THIS FORMULAE AND TABLES BOOKLET AT THE END OF THE EXAMINATION.

SCHOLARSHIP MATHEMATICS WITH CALCULUS – USEFUL FORMULAE

ALGEBRA

Quadratics

If $ax^2 + bx + c = 0$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Logarithms

$y = \log_b x \quad \Leftrightarrow \quad x = b^y$

$\log_b(xy) = \log_b x + \log_b y$

$\log_b\left(\dfrac{x}{y}\right) = \log_b x - \log_b y$

$\log_b(x^n) = n\log_b x$

$\log_b x = \dfrac{\log_a x}{\log_a b}$

Complex Numbers

$$\begin{aligned} z &= x + \mathrm{i}y \\ &= r\,\mathrm{cis}\,\theta \\ &= r(\cos\theta + \mathrm{i}\sin\theta) \end{aligned}$$

$$\begin{aligned} \bar{z} &= x - \mathrm{i}y \\ &= r\,\mathrm{cis}(-\theta) \\ &= r(\cos\theta - \mathrm{i}\sin\theta) \end{aligned}$$

$r = |z| = \sqrt{z\bar{z}} = \sqrt{(x^2 + y^2)}$

$\theta = \arg z$

where $\cos\theta = \dfrac{x}{r}$

and $\sin\theta = \dfrac{y}{r}$

De Moivre’s Theorem:
If n is any integer then
$(r\,\mathrm{cis}\,\theta)^n = r^n\mathrm{cis}(n\theta)$

COORDINATE GEOMETRY

Straight Line

Equation $y - y_1 = m(x - x_1)$

Circle

$(x-a)^2 + (y-b)^2 = r^2$

has a centre (a,b) and radius r

Parabola

$y^2 = 4ax$ or $(at^2, 2at)$

Ellipse

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ or $(a\cos\theta, b\sin\theta)$

Hyperbola

$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ or $(a\sec\theta, b\tan\theta)$

asymptotes $y = \pm\dfrac{b}{a}x$

CALCULUS

Differentiation

$y = f(x)$	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = f'(x)$
$\ln x$	$\dfrac{1}{x}$
e^{ax}	$a\mathrm{e}^{ax}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \ \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \ \cot x$
$\cot x$	$-\operatorname{cosec}^2 x$

Integration

$f(x)$	$\int f(x)\ \mathrm{d}x$
x^n	$\dfrac{x^{n+1}}{n+1} + c$
$\dfrac{1}{x}$	$\ln\lvert x\rvert + c$
$\dfrac{f'(x)}{f(x)}$	$\ln\lvert f(x)\rvert + c$

Product rule

$(f.g)' = f.g' + g.f'$ or if $y = uv$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = u\dfrac{\mathrm{d}v}{\mathrm{d}x} + v\dfrac{\mathrm{d}u}{\mathrm{d}x}$

Quotient rule

$\left(\dfrac{f}{g}\right)' = \dfrac{g.f' - f.g'}{g^2}$ or if $y = \dfrac{u}{v}$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Composite Function or Chain Rule

$\left(f\left(g\right)\right)' = f'\left(g\right).g'$

or if $y = f(u)$ and $u = g(x)$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}u}\dfrac{\mathrm{d}u}{\mathrm{d}x}$

Volume of Revolution

$y = f(x)$ between $x = a$ and $x = b$

rotated about the x-axis

Volume $= \int_a^b \pi y^2\ \mathrm{d}x$

NUMERICAL METHODS

Trapezium Rule

$$\int_a^b f(x)\,\mathrm{d}x \approx \frac{1}{2}h\left[y_0 + y_n + 2(y_1 + y_2 + ... + y_{n-1})\right]$$

where $h = \dfrac{b-a}{n}$ and $y_r = f(x_r)$

Simpson's Rule

$$\int_a^b f(x)\ \mathrm{d}x \approx \frac{1}{3}h\left[y_0 + y_n + 4(y_1 + y_3 + ... + y_{n-1}) + 2(y_2 + y_4 + ... + y_{n-2})\right]$$

where $h = \dfrac{b-a}{n}$, $y_r = f(x_r)$ and n is even.

TRIGONOMETRY

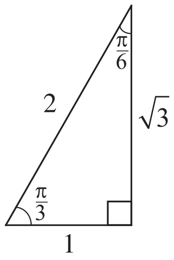
$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

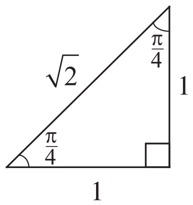
$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$





Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Identities

$$\cos^2\theta + \sin^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$$

General Solutions

If $\sin\theta = \sin\alpha$ then $\theta = n\pi + (-1)^n\alpha$

If $\cos\theta = \cos\alpha$ then $\theta = 2n\pi \pm \alpha$

If $\tan\theta = \tan\alpha$ then $\theta = n\pi + \alpha$

where n is any integer

Compound Angles

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

Double Angles

$$\sin 2A = 2\sin A\cos A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

Products

$$2\sin A\cos B = \sin(A + B) + \sin(A - B)$$

$$2\cos A\sin B = \sin(A + B) - \sin(A - B)$$

$$2\cos A\cos B = \cos(A + B) + \cos(A - B)$$

$$2\sin A\sin B = \cos(A - B) - \cos(A + B)$$

Sums

$$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$$

$$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$$

MEASUREMENT

Triangle

area $= \frac{1}{2}ab\sin C$

Trapezium

area $= \frac{1}{2}(a+b)h$

Sector

area $= \frac{1}{2}r^2\theta$

arc length $= r\theta$

Cylinder

volume $= \pi r^2 h$

curved surface area $= 2\pi rh$

Cone

volume $= \frac{1}{3}\pi r^2 h$

curved surface area $= \pi rl$ where $l =$ slant height

Sphere

volume $= \frac{4}{3}\pi r^3$

surface area $= 4\pi r^2$